

## TURBULENCE MODELING OF GAS-SOLID SUSPENSION FLOWS \*

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**INTRODUCTION**

Gas-solid two-phase flows occur commonly in many natural and industrial situations. Examples are blood flows, rocket exhaust plumes, pulverized coal gasification and combustion, and sediment transport by air and water. These flows are invariably turbulent and are characterized by the mutual coupling between the solid particles and the gas phase. Contrary to passive additives in a single-phase flow, the particles will change the flow structure of the carrying fluid. Globally, metering and heat transfer data [1,2,3] of two-phase flows shows discrepancy from the single-phase data. Further, small scale turbulence structures are also affected. Solid particles may attenuate the spreading rate and damp the turbulence intensity in a jet flow [5,6]. The alternation of the turbulence structure was found to depend on the particle size, the solid loading ratio as well as the physical properties of the different existing phases.

In general, a complete theoretical treatment of two-phase flows is not possible because of the lack of detailed understanding of the physical processes involved [7]. Previous analytical studies have not been very successful, due in part to a lack of knowledge about the turbulent flow field of the conveying gas which is a prerequisite to the solution of the two-phase flow problem. Difficulties in theoretical analysis also arise from the coupling between the two phases, i.e., the exchange of momentum, mass and energy between phases. These coupling phenomena comprise a very complex interaction which affects both the gas and particulate phases. Consideration of the infinite variety of interfacial geometries and flow regimes, various forms of non-equilibrium, and aggregation of particles complicates the problem even further.

The inability of the theoretical analysis to account for all the complicated interactions in two-phase flows is similar in the study of single-phase turbulent flows two decades or so ago. An exact theory of turbulence did not (and still does not) exist; however, using a combination of theoretical equations, modeling assumptions, and experimental evidence, mathematical models describing certain features of the flow were developed. The field of turbulence modeling has subsequently been developed to the point where single-phase turbulent flow fields can be predicted rather well using a variety of turbulence models of varying complexity [8,9]. These advances suggest that a similar combination of theory, experiment, and modeling could be used to develop computational models capable of predicting two-phase flows. However, extra sets of equations and correlations need to be formulated and modeled for turbulent gas-solid flows. The purpose of this

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paper is to discuss and review the recent advances in two-phase turbulence modeling techniques and their applications in various gas-solid suspension flow situations. In addition to the turbulence closures, heat transfer effect, particle dispersion and wall effects are partially covered here.

### APPROACHES AND GOVERNING EQUATIONS

Because of the intrinsic, complex coupling between different species in two-phase flows, there seems to be no "unified" set of governing equations that can completely describe the flow field of two-phase media. However, there are quite a number of different formulations in the literature from which to begin. One approach, the so called "discrete" or "tracking" approach, starts with an equation of motion for a single discrete particle in a turbulent fluid flow field and the particle's trajectory is calculated. For particles much smaller than the smallest scales (say Kolmogorov's microscale) of turbulent motion and for which the solid's material density is much greater than the conveying gas, the BBO equation (Basset, Boussinesq, Oseen), which is the momentum equation of a single particle, can be reduced to [10]:

$$\frac{d}{dt} v_i = \frac{1}{t_*} (u_i - v_i) + g_i \quad (1)$$

Because the Eulerian velocity  $u_i$  is a stochastic quantity when the conveying gas flow is turbulent, this simple looking ODE cannot be solved analytically due to its inherent nonlinearity.

However, progress has been made using this approach in conjunction with the turbulence closure models which have been developed for single phase flows. The basic strategy is to use the turbulence model to calculate the fluid flow field assuming that no particles are present. This calculation is used to generate the velocity in equation (1) after making suitable assumptions regarding turbulent time scales, length scales and isotropy. To account for the mutual coupling (or the "two-way" coupling [11]) of mass, momentum, and energy between phases, the extra source terms generated by particles must be included in the Eulerian sets of governing equations for the gas phase. In the mean flow fields, this can be achieved by the iterative PSIC (particle-source-in-cell) technique developed by Crowe and co-workers [12,13] or by the non-iterative, transient numerical scheme of Dukowicz [14]. The discrete particle approach can also be extended to account for the particle-turbulence interactions which have two aspects -- the turbulent particle dispersion (the influence of fluid turbulence on the particles), and the "modulation" effect [15] (the effect of particles on fluid flow turbulence). These will be discussed in further detail in the next section.

In the non-discrete (continuum) approaches, two formulations are commonly used; the first considers the gas-solid suspension to be represented as a single inhomogeneous medium. The interactive forces between the phases are taken account of by internal stresses which must be related by constitutive equations to the bulk properties of the medium. Sets of governing equations for this approach were first formulated by

Barenblatt [16] and described in detail in Monin and Yaglom [17]. This approach was also used recently in heat transfer analysis of a gas-particle pipe flow [18].

The other approach is the so-called "two-fluid" approach. This approach regards the gas and particles as two inter-penetrating continua in much the same way as the two species of a flowing binary mixture, for example. Here, the cloud of particles is regarded as a continuum and the governing equations are obtained by properly averaging the conservation equations over a volume and expressing the equations in differential forms. Many authors, namely Murray [19], Drew [20], Marble [21] and Ahmadi [22], have described the two-phase flow based on the two-fluid formulation and applied it to some physical processes. It is often not possible to formulate a general set of governing equations for gas-solid two-phase turbulent flows due to the lack of understanding and differences in interpretation of the physical processes involved (for example, the "solid-phase pressure" term [23]). In order to obtain theoretical relations of two-phase turbulent flows, several assumptions have to be invoked to simplify the formulation. These are:

1. The particle phase is dilute (volume fraction of particles,  $\phi \ll 1$ ) and is made up of particles spherical in shape and uniform in size. The particle material density  $\rho_p \gg \rho$ , so that the model is valid when  $\rho_p = O(\rho)$ . This assumption is required because we ignore particle-particle collisions, the frequency of which increase quadratically with loading. The uniformity of particle size reduces the book-keeping in the formulation; extension to poly-dispersed non-uniform size distribution is a straight forward matter for dilute suspensions.
2. Both the particulate and fluid phases behave macroscopically as continua. The fluid phase is Newtonian and both phases have constant physical properties and do not undergo any phase change. The continuum hypothesis assumes that the mathematical "points" are large enough to contain many particles and fluid molecules to ensure a stationary average. In order to satisfy the "dilute suspension" and continuum assumptions simultaneously for particle phases, some stringent restrictions regarding the number of particles in a smallest control volume made up of Kolmogorov microscale, the distance between particles to avoid direct inter-particle interactions, have been discussed [10, 24, 25]. However, the continuum approach has proven to be applicable also to situations which do not strictly meet such conditions [26].
3. The mean flow is steady and incompressible. Molecular diffusion, Brownian motion and gravity effects on the particulate phase are negligible compared with turbulent diffusion. Electrical and magnetic forces are not considered here.

With the above assumptions, we may adopt the governing equations developed by Marble [21, 27] and Hinze [25] which are applicable to dilute gas-particle flows. Marble used statistical averages for the particle cloud and postulated the macroscopic governing equation for the gas phase. Continuity equations are written for each phase:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) = 0 \quad (2)$$

$$\frac{\partial \rho_p}{\partial t} + \frac{\partial}{\partial x_i} (\rho_p v_i) = 0 \quad (3)$$

Here  $\rho_p$  is the mass of particles per unit volume of mixture (or "density" of the particulate phase where  $\rho_p = \rho_s \phi$ ,  $\phi$  is the particulate phase volume fraction). The momentum conservation equations for each phase are:

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_j} (\tau_{ij}) + F_{pi} \quad (4)$$

$$\rho_p \frac{\partial v_i}{\partial t} + \rho_p v_j \frac{\partial v_i}{\partial x_j} = - F_{pi} \quad (5)$$

Here  $F_{pi}$  represents the force acting on the primary fluid per unit volume due to the presence of the particle. Note that due to dilute assumption, the multiplication of  $(1 - \phi)$  by each term in equation (2) and (4) was replaced by 1. Of special note is that the values of  $u_i$ ,  $\rho$ ,  $p$  that appear in the continuum relations (2) and (4) are, in a sense, "smoothed" variables. The detailed gas disturbance caused by the particle motions are omitted from the instantaneous gas velocity vector  $u_i$ . Since the gas velocity varies strongly in the neighborhood of a particle that is moving through the gas, use of these smoothed variables in continuity, momentum and energy relations requires that all particle wakes or regions of immediate influence are dissipated very rapidly over the gas control volume. Hinze [25] treats this problem by attributing the forces around the particle as the external forces and disregarding the modified velocity field around the particles. If this external drag force follows Stokes law, then the fluid velocity  $u_i$  in the Stokes drag law is at "infinity", i.e., a large distance from the particle center so that the detailed fluid motion in the neighborhood of the particle is still not accounted for. However, the inadequacy of this model is not important for small volume fractions of particles having a not too large velocity relative to the gas. But for large volume fractions and cases in which particles may form into groups by trailing another in its wake, large regions of the flow may be inadequately modeled (c.f. [25] and [28]).

Several derivations concerning the two-fluid model equations have appeared in the literature. The derivations include those of Hinze [29], Soo [30], Drew and Segel [31], Ishii [32], Nunziato [33] and more recently, Roco and Shook [34]. The resulting equations differ in various ways such as the pressure gradient term for both gas and particulate phase, momentum source term, and shear stress tensor of the secondary phase although general constitutive equations relating stress and flow properties have not yet been developed. However, for the low concentration limit of suspension flows of small spherical particles, most of the derivations will recover similar forms. For example, the theory proposed by Ahmadi [35] which was general to the extent that it could be applied to both concentrated and dilute two-phase flows could be shown to recover the theory of dusty gas as derived by Saffman [36] in the low solid volume fraction range. The general expression for the internal forces between solid and continuous phase is discussed by Truesdell and Toupin [37] (also see [38]) and Drew and Segel [31]. The philosophical

reasons for using the two-fluid continuum approach and the common feature of dispersed two-phase flow systems can be seen in Drew's [20] review paper.

By performing the Reynolds decomposition and time averaging of equations (2) - (5), the following mean equations for statistically steady flows result.

$$\frac{\partial \bar{U}_i}{\partial x_i} = 0 \quad (6)$$

$$\frac{\partial \bar{\rho}_p V_i}{\partial x_i} = - \frac{\partial}{\partial x_i} \bar{\rho}_p' v_i' \quad (7)$$

$$\rho U_j \frac{\partial \bar{U}_i}{\partial x_j} = - \frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 \bar{U}_i}{\partial x_j \partial x_i} - \frac{1}{\rho} \frac{\partial}{\partial x_j} \bar{u}_i' u_j' + F_{p,i} \quad (8)$$

$$\bar{\rho}_p V_j \frac{\partial \bar{V}_i}{\partial x_j} = - \frac{\partial \bar{\rho}_p' v_i'}{\partial x_j} \frac{\partial \bar{V}_i}{\partial x_j} - \frac{\partial}{\partial x_j} (\bar{\rho}_p' \nabla_i' \nabla_j') - \frac{\partial}{\partial x_j} (v_j \bar{\rho}_p' v_i') - F_{p,i} \quad (9)$$

where some use of the continuity equations has been made in deriving equations (8) and (9). As a consequence of dilute suspension, triple correlations involving fluctuations in the particulate phase density are considered negligible. At this point the mean interaction term  $\bar{F}$  needs to be specified. Empirical expressions for the interaction terms  $F_{p,i}$  have been summarized by [39] for low and moderate solids concentrations. The appropriate relationships are given by

$$\begin{aligned} \bar{F}_{p,i} &= \frac{1}{t_*} (1 + 0.179 \sqrt{\text{Re}_p} + 0.013 \text{Re}_p) \bar{\rho}_p (V_i - U_i) \quad (10) \\ &+ \frac{1}{t_*} (1 + (3/2) 0.179 \sqrt{\text{Re}_p} + 0.113 \times 2 \text{Re}_p) \bar{\rho}_p' (v_i' - u_i') \end{aligned}$$

where

$$\text{Re}_p = \frac{|U_i - V_i| d_p}{\nu} \quad (11)$$

$$\text{and } t_* = d_p^2 \rho_s / 18 \rho \nu \quad (12)$$

We note that several turbulent correlations appear in equations (7) - (9) in addition to the conventional Reynolds stress  $\bar{u}_i' v_i'$  for the single-phase flows. These terms arise from the velocity fluctuations and fluctuosity volume fractions of the particulate phase and represent the turbulent momentum flux and mass flux of particles. To close the set of governing mean equations, models are required for these second order

correlations. The field of turbulence modeling for single-phase flows is a rapidly expanding one and will form the two-phase closure models described here.

## TWO-PHASE TURBULENCE MODELING

The hierarchy of turbulence closure models has been received by Reynolds [40], and recently in [8,9]. The proposals range in complexity from zero equation models where the turbulent fluxes are modeled as if they were molecular fluxes, with an eddy diffusivity related to mean flow structures to Mean Reynolds Stress models where separate transport equations are solved for each component of the turbulent flux vectors and tensors. Most two-phase turbulence models follow the single-phase turbulence models for incompressible flows closely; their modeling is discussed in the following.

The most common and simplest modeling technique is to assume a Newtonian type constitutive equation for relating the turbulent fluxes to the mean field through an eddy viscosity. For gas phase Reynolds stresses,

$$\overline{u_i'v_j'} = - \nu_f S_{ij} + 2/3 \delta_{ij} k \quad (13)$$

where  $S_{ij}$  is the mean rate of strain tensor of gas flows  $\frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$

$$k = 1/2 \overline{u_i'u_i} \quad \text{and} \quad \nu_f \quad \text{is the eddy viscosity.}$$

Depending on the level of complexity employed, the eddy viscosity could be specified by zero-equation mixing length models, one-equation models or two-equation models. Due to the presence of solid particles, the eddy viscosity constructed by these models must take into account this effect.

## ZERO-EQUATION MODELS

Early theoretical studies [41 - 43] indicate that the presence of solid particles decreases the eddy viscosity of the gas flows arising from dissipation of turbulence energy at the interface between solid particles and the fluid. These results lead several first-order closure schemes which modify the eddy viscosity for the clean gas flow without suspension of solid particles,  $\nu_{f0}$ . For example, Owen [41] proposed

$$\frac{\nu_f}{\nu_{f0}} = \left( 1 + \frac{\bar{\rho}_p}{\rho} \right)^{-1/2} \quad (14)$$

for the case  $t_*/t_e \leq 1$  and

$$\frac{\nu_f}{\nu_{f0}} = \left[ 1 + \left( \frac{\bar{\rho}_p}{\rho} \right) \left( \frac{t_e}{t_*} \right) \right]^{-1/2} \quad (15)$$

for  $t_e/t_* \geq 1$

This model has been used by Melville and Bray [44] for application in a turbulent free jet of dilute gas-particle mixture and has been further modified by Choi and Chung [45] and Chung et. al [46] for application in a wall-bounded shear flow. Most closure models developed at this level heavily involve empirical information and limiting case (loading ratio approaching zero) analysis and, in most cases, ignore the effect of particle size [47,48]. This level of models are very useful in engineering analysis because of their simple forms. However, they fail to handle some important effects, such as the "turbulence modulation", and besides, it is hard to prescribe the "mixing length" and the effect of particle on the mixing length scale. The next level of models, which incorporate a transport equation for the turbulence kinetic energy, and thus the velocity scale, were developed in the hope of providing additional generality and at the same time account for the effect of particles on the turbulence structure.

#### ONE-EQUATION MODEL

An equation describing the dynamics of the gas-phase turbulence kinetic energy can be derived from equations (4) and (8) by simple manipulations. For statistically steady, high Reynolds number flows it is given

$$U_i \frac{\partial k}{\partial x_i} = - \frac{\partial J_i}{\partial x_i} + P_k - \epsilon + \overline{u_i' F_{pi}} \quad (16)$$

Here  $P_k = - \overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j}$  is the rate of production of turbulence energy and

$\epsilon = \nu \frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j}$  is the rate of energy dissipation rate.

$$J_i = \frac{1}{2} \left( \frac{\overline{u_i' u_j' u_j'}}{\overline{u_i' u_j'}} + \frac{\overline{p' u_i'}}{\rho} - 2\nu \frac{\partial k}{\partial x_i} \right) \quad \text{and}$$

$\overline{u_i' F_{pi}}$  the extra particle production (or dissipation) term, all per unit of mass. Probably the first attempt to use a one-equation turbulence model to study the two-phase flows is that of Dannon et. al [49]. They applied a  $k-\ell$  closure model to a particle-laden axi-symmetric jet. The length scale  $\ell$  was specified algebraically and was taken to be the same as that of a single-phase jet. For the  $k$ -equation (16), the diffusion term and production term were modeled following the conventional single-phase gradient-type modeling technique. In their study, quasi-equilibrium (i.e.  $U_i \approx V_i$ ) and mono-dispersed particles in Stokes regimes were assumed,

which simplify the interaction terms  $F_{pi} = \frac{\rho_p(v_i - u_i)}{t_*}$  and the turbulence kinetic energy equation becomes

$$U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{v_f}{\delta_k} \frac{\partial k}{\partial x_i} \right) + v_f \left( \frac{\partial U_i}{\partial x_j} \right)^2 - C_D \frac{k^{3/2}}{l} + \frac{\rho_p}{\rho} \frac{(\bar{u}'_i v'_i - \bar{u}'_i u'_i)}{t_*} + \frac{1}{\rho} \frac{\rho_p u'_i (v'_i - u'_i)}{t_*} \quad (17)$$

The triple correlation was neglected and the concern was the modeling of the additional dissipation term created by the particle slip velocity at the fluctuation level. This term is similar to the turbulence "modulation" effect attributed to the inability of dispersed-phase particles to completely follow turbulent eddy fluctuations at high frequency. This added dissipation mechanism has been experimentally observed [2, 5, 6, 50, 51] and has gained much attention in recent two-phase modeling studies.

The fluctuating velocity correlation of this term is bounded by

$$0 \leq -(\bar{u}'_i v'_i - \bar{u}'_i u'_i) \leq 2k \quad (18)$$

where the two bounds represent the cases where particles completely follow the fluid ( $\bar{u}'_i = v'_i$ ) and stationary particles relative to the velocity fluctuation ( $v'_i = 0$ ). Dannon et. al [49] proposed a model that has the correct limiting behavior

$$- (\bar{u}'_i v'_i - \bar{u}'_i u'_i) = 2k [1 - \exp(-\beta(t_*/\tau))] \quad (19)$$

where  $\tau = (v/\varepsilon)^{1/2}$  is the Kolmogorov time scale, and  $\beta$  is a model constant.

The use of the time scale  $\tau$  was argued [52] to be inappropriate since the eddies contributing most to the correlation  $\bar{u}'_i u'_i$  are the energetic eddies which have an integral time scale  $t_e$ . Dannon et. al [49] indicated that this model did not give good prediction due to a change in the structure of the turbulence and the structure was represented by the length scale. As a result, they had to arbitrarily modify the production and dissipation terms to reflect the structural variations. Due to the difficulty of specifying the length-scale distribution a priori in a flow and appropriate modeling for particle effect, most workers have abandoned one-equation models in favor of two-equation or even stress-equation models in which the length scale is computed from a transport equation.

## TWO-EQUATION MODELS

Most studies in two-phase turbulence modeling utilizing a transport equation for the turbulence length scale  $\lambda'$  are based on a modeled equation of the isotropic dissipation rate  $\epsilon$ ; this equation can also be derived from equations (2), (4), (8) by appropriate differentiation, multiplication and averaging. The exact  $\epsilon$ -equation consists of 67 terms with particle's effect accounted for [53]. For high Reynolds number flows and based on an order-of-magnitude analysis [54], the groups representing the production of  $\epsilon$  by vortex stretching, the viscous destruction of  $\epsilon$ , and the diffusive flux of  $\epsilon$  in the  $X_i$  direction, which are not affected by particles, are usually modeled following the single-phase  $k-\epsilon$  model of Jones and Launder [55]. The resulting modeled equation (except the extra particle destruction term) becomes

$$\begin{aligned} \frac{\partial}{\partial x_j} (U_j \epsilon) &= \frac{\partial}{\partial x_j} \frac{\nu_t}{\delta \epsilon} \frac{\partial \epsilon}{\partial x_j} - C_{\epsilon 1} \overline{u_i' u_j} \cdot \frac{\partial U_i}{\partial x_j} \frac{\epsilon}{k} - C_{\epsilon 2} \frac{\epsilon^2}{k} \\ &+ 2 \nu \frac{\overline{\partial u_i' / \partial x_k} \overline{\partial F_{pi}' / \rho_p}}{\partial x_k} \quad (20) \end{aligned}$$

The last term in the RHS of the above equation is the contribution from inter-phase transport and is another main effort in two-phase modeling. This equation is solved simultaneous with the  $k$ -equation to estimate the eddy viscosity  $\nu_t = C_\mu k^2 / \epsilon$ . Since the effects of particles are modeled through the  $k$  and  $\epsilon$  equations,  $C_\mu$  is assigned 0.09 same as the single phase flows.

In the two-equation model level, there have been several proposals for modeling the extra terms in the  $k$  and  $\epsilon$  equation. Chen and Wood [52] basically followed [49] and proposed exponential forms for added dissipation terms in both the  $k$  and  $\epsilon$  equation. In the  $k$  equation, the correlation

$\overline{u_i' v_i'}$  is modeled as

$$\overline{u_i' v_i'} = 2k \exp(-B_k t_*/t_e) \quad (21)$$

where  $t_e$  is the time scale of the energetic eddies and in the context of the model is given by  $k$  and  $\epsilon$ .  $t_e$  here has often been interpreted as the lifetime of a typical turbulence eddy by [56,57] in their Lagrangian calculations. The time ratio  $t_*/t_e$  is the Stokes number [12] measuring the response of how quick the particle responds to a typical eddy turnover. To generalize the model, the constant  $B_k$  was introduced and was determined by limiting behavior for small particles, which corresponded to the linear perturbation analysis with respect to a passive additive by [42]. A similar approach has been used by Pourahmadi and Humphrey [58] and Gavin et. al [59] in the  $k$ -equation. Their model for this term is summarized:

$$\overline{u_i' v_i'} = 2k / (1 + t_*/t_e) \quad (22)$$

It can easily be shown, for small values of  $t_*/t_e$ , that this model yields the same results as equation (21), depending on the numerical value of  $B_k$  and the form assumed for  $t_e$ . Genchev and Karpuzov [60] assumed  $t_* \gg t_e$  so that

$\overline{u_i'v_i'} = 0$  and no model was required. However they also assume that the particles will follow the mean fluid motion which implies  $t_* \ll t_*$ . This inconsistency plus the lack of model comparisons with data in their paper casts doubt on their model.

The model of Elgobashi and Abou-Arab [53] for the correlation  $\overline{u_i'v_i'}$  is based on Chao's [61] solution of the linearized Lagrangian equation of motion of a spherical particle in a turbulent flow. Their model, in its most general form, is extremely difficult to implement, especially for wall-bounded two-phase flows, owing to the necessity of computing definite integrals over all possible frequencies of fluid motion at every grid point. Given the uncertainties of the model it seems more appropriate to use a relatively simple model which exhibits the correct asymptotic behavior such as [52] and [58] (c.f. [62]).

As in modeling single-phase flows it is the  $\epsilon$ -equation which provides the greatest uncertainty. In the model of Chen and Wood [26], particle-hydrodynamic drag force was assumed to follow Stokes law, thus the last term of equation (20) became

$$2\gamma \frac{\overline{\rho_p}}{\rho} \frac{\partial \overline{u_i'}}{\partial x_k} \frac{\partial F_{pi'}}{\partial x_k} = \frac{2}{t_*} \frac{\overline{\rho_p}}{\rho} \left[ \nu \frac{\partial \overline{u_i'}}{\partial x_j} \left( \frac{\partial \overline{v_i'}}{\partial x_j} - \frac{\partial \overline{u_i'}}{\partial x_j} \right) \right] \quad (23)$$

A similar exponential model was proposed for this term as given by equation (19). A different time scale, i.e. Kolmogov's time scale  $\tau$ , was used here in place of  $t_*$  since the eddies contributing most to the high frequency destruction mechanisms are the dissipative eddies which have a time scale

$(\nu/\epsilon)^{1/2}$  [ $= \tau$ ]. In [26] it was assumed that  $\frac{\partial \overline{u_i'}}{\partial x_j}$  and  $\frac{\partial \overline{v_i'}}{\partial x_j}$  are completely uncoupled on this time scale since  $t_* \gg \tau$  in most practical gas-solid turbulent flows. In this limit equation (23) becomes  $(2\epsilon/t_*)(\overline{\rho_p}/\rho)$

Clearly this model is not correct for very small particles or if  $t_* \approx \Theta(\tau)$ . In the model of [58] an additional term is added but it assumes that the added sink of dissipation to be a function of the integral time scale  $t_*$ , and hence does not seem to be particularly appropriate. The extra sink and destruction of  $\epsilon$  are modeled collectively by [53] as  $C_{\epsilon_2} \epsilon_t (\epsilon/t)$  in equation (20) where  $\epsilon_t$  equals  $\epsilon$  plus the extra dissipation terms appearing in the  $k$ -equation and  $C_{\epsilon_2}$  is kept the same constant as in the single-phase flows.

#### DISPERSION OF SUSPENDED SOLID PARTICLES

Extremely small particles which behave like trace molecules can be treated as "passive" contaminants in the turbulent flow field. The behavior of clouds of particles may be extended from single-particle dynamics when the mixture is very dilute, say, the volume fraction of solid  $< \Theta(10^{-2})$  (c.f. [7]). The subject of passive additive transport has been treated extensively in the text of Monin and Yaglom [17]. See also the book by Hinze [63] and the review paper by Launder [64]. However, as the particle size increases, dispersion will be opposed by particle inertia and so once some critical particle size is exceeded, discrete particle dispersion must be treated in a different way from "passive" contaminant diffusion.

For dilute suspensions, the particle trajectories can be calculated by the tracking approach. In this approach, particle dispersion due to turbulence has been modeled by random walk [65] or a Monte-Carlo Stochastic method [14,66]. These methods usually require extensive computational storage and time to achieve a stationary average. In some Lagrangian approaches, certain types of diffusional velocity have been modeled for the particle motion which is usually proportional to the concentration gradient [67,68]. In the two-fluid approach, particle dispersion due to turbulence is represented by the correlations and/or . In lower level closures, these are usually modeled as a gradient type, Fickian diffusion process:

$$\overline{\rho_p' u_i} = D_t \frac{\partial \overline{\rho_p}}{\partial x_i} \quad \text{and} \quad \overline{\rho_p' v_i} = D_p \frac{\partial \overline{\rho_p}}{\partial x_i} \quad (24)$$

This constitutive equation is arbitrary at this point; however it may be justified theoretically under certain conditions [17].

Values of  $D_t$  and  $D_p$  are calculated by the value of eddy viscosity  $\nu_f$  in most models by introducing the turbulent Schmidt numbers. Thus  $D_t = \nu_f / Sc_t$  and  $D_p = \nu_p / Sc_p$ , or  $D_p = \nu_p / Sc_p$ , where  $\nu_p$  is the effective eddy viscosity of the dispersed phase (to be discussed later). This type of phenomenological approach for diffusion process heavily relies on the classical theory of "fluid point" diffusion of Taylor (c.f. [63]). However, when particle size increases, discrete particle diffusion is opposed by particle inertia and the crossing-trajectories effect [69,70]. Since heavier particles have the tendency to "fall out" from one eddy to another, the correlations between particles and fluid velocities decrease. The effect is to decrease the particle dispersion. The effect of the particle inertia is not that clear. The inertia effect is characterized by the particle relaxation time  $t_*$ , which is controlled by the physical properties of particles and the fluid, and the flow characteristics. There have been arguments concerning the characteristic flow time scales [71,72,73] for turbulent dispersion, although it has been indicated that the diffusivity of the heavy particles is a little larger than that of the light particles. Higher-order modelings such as the one developed by [74] are able to predict this behavior. In the context of the lower-order phenomenological technique just mentioned, the Schmidt number should be modeled taking into account these effects. Recently one such model has been developed including a constant drift velocity [75]

$$D_p = 1 / (1 + 0.3 |U_i - V_i|^2 / \overline{v_j' v_j})^{1/2} \quad (25)$$

in which the coefficient 0.3 was tuned based on the lateral dispersion of solid particles and measurements of [71,73].

However, in most models [26,53,58] the turbulent Schmidt number was simply set to some constant value following the turbulent mass transfer of a passive additive [76]. Some experimental data for gas-solid jets [77] however indicate that a constant value of  $Sc_t$  (i.e. independent of loading) is appropriate. For axi-symmetric flows,  $Sc_t = Sc_p = 0.7$  is used [52,78]. In [53,58], the turbulent Schmidt number was simply chosen to be one.

Finally, due to the continuum formulation, the correlation  $\overline{v_i'v_j'}$  has to be modeled. Following the gradient type model for the gas phase, this turbulent stress in the particulate phase is modeled from the Boussinesq assumption:

$$\overline{v_i'v_j'} = - \nu_p \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) + \frac{2}{3} \delta_{ij} \left( k_p + \nu_p \frac{\partial v}{\partial x} \right) \quad (26)$$

Earlier analytical studies [79,80] have contributed to the understanding of some of the basic mechanisms of indirect interaction between particles through the surrounding particles. This has lead Melville and Bray [44] in their zero-equation modeling to propose

$$\frac{\nu_p}{\nu_t} = 1 / (1 + t_* / t_e) \quad (27)$$

Similar models have been used by [46,52] although the evaluation of  $t_e$  is somewhat different. Alonso [81] reviewed some developments in determining  $\nu_p$  and recommended the use of Peskins [82] formula

$$\frac{\nu_p}{\nu_t} = 1 - (T_L^2 \epsilon / 15\nu) (3K^2 / (K + 2)) \quad (28)$$

when  $K = 2t_*/T_L$  and  $T_L \approx k/\epsilon$ . This model has been used by [58,83]. Although this model recovers the correct form in the limit  $t_* \rightarrow 0$  (i.e.  $\nu_p = \nu_t$ ) as equation (27), it will yield negative value of  $\nu_p$  for reasonable values of  $t_*$ ,  $\tau$  and  $T_L$  taken from pipe flow data. It is not likely that  $\nu_p < 0$  is physically appropriate, casting doubt on this model.

The modeling technique discussed above based on the continuum approach, especially the modification of  $k$  and  $\epsilon$  equation, has been extended and adopted in some Lagrangian formulations. It has been shown by Shuen et. al [6] that using the stochastic formulation instantaneous properties are known; therefore, the extra dissipation term due to particles in the  $k$ -equation is exact and requires no modeling. This calculation is rather complex and recently Mostafa and Mongia [75,91] have utilized the continuum two-phase model of [58] to model this term in Lagrangian calculations. A similar approach has been taken by [84] which highly simplified the stochastic calculations. The extra sink term in the  $\epsilon$ -equation is not closed in Monte-Carlo stochastic formulation and is modeled through a gradient type model in [6]. The sensitivity of this term has been tested recently [85] and it has been found that this term is important in conjunction with the modulation term in the  $k$ -equation. Incorporation of the effects of particle on turbulence scale and the response of the dispersed phase in two-phase turbulence models is essential for representing the structure of particle-laden turbulent flows.

## WALL EFFECTS

For wall-bounded flows, boundary conditions for both gas and particulate phases are required. This is particularly important when other transport processes such as heat transfer and erosion are involved. The effect of particles on boundary layer and viscous sublayer flows has been studied analytically [10,86]. In most calculations, it is assumed that the influence of the particulate phase on the velocity defect law is to modify the logarithmic law in sublayer. Based on the Monin-Obakhov similarity analysis for the analogous stable stratified atmospheric boundary layer, a set of "wall functions" taking into account the effects of particle size and loading was used by Chen [87]. The logarithm profile was modified as

$$\frac{U}{U_*} = \frac{1}{\kappa} \ln [y^+] + \beta \frac{\bar{\rho}_p}{\rho} R_f \quad (29)$$

with  $\beta \approx 5$  and particulate flux Richardson number

$$R_f = \frac{\bar{u}_i \cdot F_{pi}}{\left( - \frac{\bar{u}_i \cdot u_j}{\partial x_j} \frac{\partial U_i}{\partial x_j} \right)} \quad (30)$$

The wall shear stress of the two-phase flow is related to that of the single-phase flow by (c.f. [10]).

$$\frac{\tau_w}{\tau_{w0}} = 1 + \frac{\bar{\rho}_p}{\rho} \quad (31)$$

The wall boundary conditions for particulate phase are complicated by the unsteady particle-wall interactions such as deposition of particles on the wall and re-entrainment mechanism. The resulting piece of information from a multitude of particle-wall interactions can be assessed as the slip velocity for the particulate phase at the wall [88,89]. An expression for the slip velocity and wall shear stress was suggested by Soo [88] based on rarefied gas-dynamics theories. The lack of particle-particle interaction, in accordance with the dilute suspension assumption gives a wall slip velocity:

$$v|_w = l_p \cdot \left. \frac{\partial v}{\partial n} \right|_w \quad (32)$$

and stress

$$\tau_p|_w = \frac{1}{2\sqrt{\pi}} \bar{\rho}_p v^2 \left. \frac{2}{3} \left( \frac{\bar{v}_i \cdot \bar{v}_i}{\bar{v}_i \cdot \bar{v}_i} \right)^{1/2} \right|_w \quad (33)$$

where the fluid-particle interaction length  $l_p$  is given

$$\ell_p' = [(\bar{u}_1' - \bar{v}_1')^2]^{1/2} t_*$$

and  $\bar{v}_y$  is set to be zero at the wall following an impermeable wall condition for particles. These expressions have been utilized by [58,87] for their wall-bounded calculations. The effect of the walls on the particle drag coefficient, the particle turbulent intensity and the correlation  $\bar{u}_1' \bar{v}_1'$  and  $\bar{u}_1' (\bar{u}_1' - \bar{v}_1')$ , which represent the particle-gas interaction and contribute to the modulation of the wall turbulence structures, has been studied recently by Risk and Elghobashi [90] by including Magnus force and lifting force in the particle dynamic equation. Their analysis can be incorporated for detailed gas-particle wall function development.

#### SUMMARY AND DISCUSSION

Recent developments of two-phase turbulence models were reviewed. Most existing models are constructed following the familiar form of single-phase turbulence models. As in single-phase problems, most models are addressed through classical Boussinesq gradient-type diffusion processes and scaling arguments. Most models are also developed based on the treatment of turbulent suspensions in the context of the continuum, two-fluid theory of mixtures.

The appeal of the two-phase closure technique embedded in the two-fluid continuum formulation is that it provides an axiomatic approach on which the analogous single-phase turbulence models are built. In practice, one is confronted with the difficulty of constructing specific constitutive models for the stresses and momentum transfer, turbulent mass fluxes and mass transport in which additional fluctuating fields are magnified by the presence of solid particles. Following the modeling approach in single-phase flows, for simple flows (such as free shear flows) most two-phase models were addressed through classical Boussinesq assumptions and characteristic scaling arguments. Depending on the relaxation time scales, particles not only influence the higher wave number end of the gas-phase turbulence spectrum [10,25], but also the energy-containing range of the turbulence spectrum which is largely responsible for mixing [72,73]. Most proposals for treating turbulence modulations based on the two-equation  $k-\epsilon$  model were not particularly successful for complex flows since they did not incorporate the turbulence scale effects and the response of the dispersed phase. Higher-order closure schemes or closures involving multiple-scale characterization of the gas turbulent spectrum are obviously called for, and some steps in this direction have been taken recently [74,87]. Additional measurements similar to that of [72,73] are also needed to gain a better understanding of particle-turbulence scale interactions and modulations in multiphase flows.

The real challenge and difficulty in developing a two-phase closure model for particle dispersion in connection with the two-fluid formulation is encountered in wall-bounded flows and poly-dispersed systems. Establishment of wall boundary conditions for the particle concentration and velocities depend on the interaction of particles with the wall. Particle-wall collisions are not always elastic and the phenomena is unsteady. The slip velocity boundary condition based on rarefied gas dynamics concepts is

probably not appropriate since the normal component of averaged velocity is not zero. Detailed measurements [92,93] and analysis [93,94] are needed. To extend the two-fluid continuum mixture theory for poly-dispersed situations, the continuous droplet models such as the one described in [95] can be used. The particles are represented by a statistical distribution function in a multi-dimensional space of droplet size, velocity location and time. The properties of particles are determined by solving the conservation of the distribution function.

One merit of the two-phase turbulence models developed on the continuum formulation is that they can be accommodated into the Lagrangian approach and do not require excessive computational storage and time [75,84]. Incorporation of more physics as turbulent combustion, evaporating sprays, boundary layer dust ingestion, poses no conceptual difficulties. Further testing and validation through well-defined experiments for more complex flows to establish the universality of the model constants are highly recommended.

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